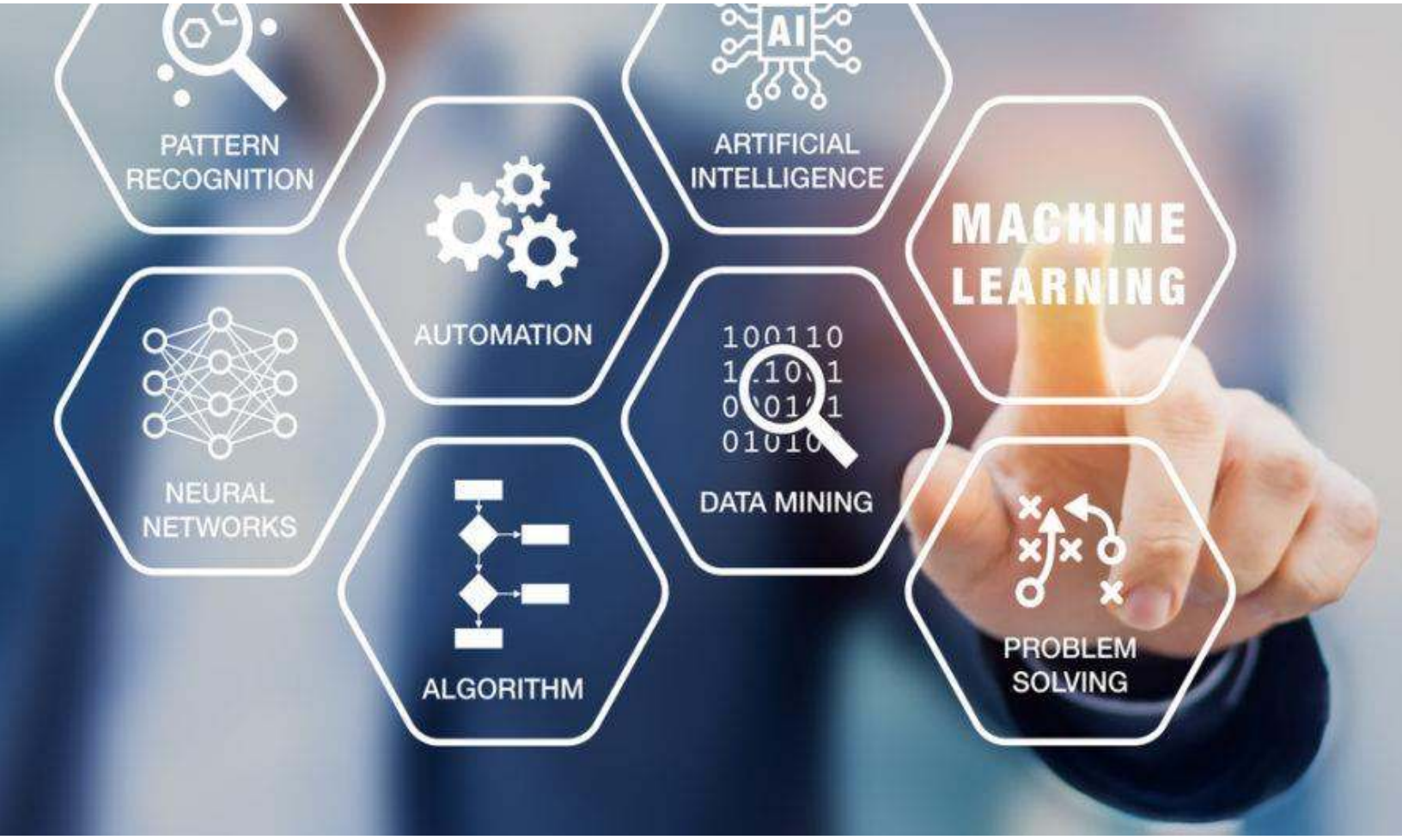
ROBO ADVISORY

BEYOND MEAN VARIANCE OPTIMIZATION



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INTRODUCTION

Robo advisors are a relatively new invention in the investing world and made headlines in recent years, online investment management services like Wealthfront and Betterment that manage your investments for you through unique algorithms. As the name suggests, a form of automated investment management service that requires little to no human interaction for investment decision making.

In many respects, a need for professional investment advice has never been greater, as investors grapple with global and geopolitical uncertainty, prolonged low and negative interest rates, and longer lifespans. Robo-advisors, automated investment platforms that provide investment advice without the intervention of a human advisor, have emerged as an alternative to traditional sources of advice.

A key component of Robo advisory service models is the use of optimization algorithms, which are designed to solve investment challenges such as portfolio allocation for a specific type of investor. The outcomes derived from algorithms used by various Robo advisor will vary based on the methodologies, assumptions, tools, and data inputs used by the algorithms. The classic Mean Variance Optimization Model is the most widely accepted technique which determines expected returns and volatilities for the asset classes or securities, along with correlations among the asset classes, and then uses a complex algorithm to determine the best weightings for each risk level.

The mean variance approach to portfolio selection, developed by Nobel laureates Harry Markowitz and James Tobin with modern portfolio theory, is the foundation of all modern day optimization models. Conventional wealth managers to Robo advisors employ mean variance optimization to structure efficient portfolios, however, it can sometimes produce highly skewed portfolios, and especially when using historical asset class returns as inputs, most of these algorithms modify their inputs or constrain the outputs. The use of Black Litterman model & adding a classic Fama-French factor tilt is a common practice to overcome the limitation of modern portfolio theory.

This paper explores a primary investment allocation framework employed by Robo advisors; the mean variance analysis along with prerequisites like data input and assumptions required to construct efficiently diversified portfolios customized for an investor based on their profile & investment objective, without any help from an active investment manager.

This being a broad subject, the research does not covers many other features, models, benefits or limitations of Robo advisory platforms. It only focuses few broad aspects of a typical Robo advisory platform such as Risk Profiling & Goal Based Investment Planning, Factor Based Investment Allocation & Portfolio Optimization etc. whose objective is to generate customize, an optimum investment recommendation for an investor based on their Social profile, Risk taking abilities, Investment goals as well as their specific views on the market.

RISK PROFILING

Investor risk profiling is at the heart of private wealth management. Be it an active or passive investment model, Risk Profiling is arguably the first and a mandatory step in investment planning. In theory, without proper knowledge of the investor's goals, time horizon, liquidity needs, and risk aversion, it is impossible to recommend suitable investments or build efficient long-term investment strategies for that investor.

Robo Advisory uses the concepts of classical decision making theories such as modern portfolio theory, and the capital asset pricing model (CAPM) which uses the risk profile an important input. It is assumed that investors are inherently risk averse and take on additional risk only if they judge that higher anticipated returns will compensate them for it. One of the fundamental results of modern portfolio theory is that, under the assumptions of the CAPM (Sharpe 1964), all investors invest in a combination of the asset with a least risk for a preferred return from their investments.

The assessment of an investor is required to the ability and willingness take financial risk. The ability can be assessed with objective economic circumstances, such as the investor's investment horizon, liquidity needs, income, and wealth, as well as tax rates and other factors. The primary distinguishing feature of risk capacity is that it is relatively immune to psychological distortion or subjective perception. On the other hand, willingness to take risk may be understood as the combination of behavioral traits and emotional responses with the risk where it can be determined by the degree of psychological or emotional pain the investor experiences when faced with a financial loss. These emotional factors are often even more important for practitioners to understand than the objective economic circumstances of the investor; yet, they are harder to measure

The risk profiler assessment can be done through questionnaire to determine both the level of risk an individual can take and willing to take. Although this approach is conventional and can be highly unreliable, it is by far the most used method across active and passive investment management. To make it more accurate, one can use various methods to make it more reliable such as consistency among her answers. The less consistent the answers, the exponentially less risk tolerant the investor is likely to be. For example, if an individual is willing to take a lot of risk in one case and very little in another, then she is inconsistent and is therefore assigned a lower risk tolerance score than the simple weighted average of her answers.

One can also give weightage to the social profile of the investor, their assets and liabilities, prediction of their future income and changes in the spending habits and life events. The selected investment goal can also be an important factor in estimating the risk tolerance. For instance, an ability to save a greater the excess income for a goal, the more risk the investor can take. Conversely if the expected income is less than the target, then the investor cannot afford to take much risk with the investments.

It is essential to reevaluate the financial profile for any changes that may affect their risk tolerance. For example getting married, having kids, benefiting from equity appreciation associated or being promoted to a significantly higher paying job can have a major impact on the risk score. In addition, one should gradually adjust clients' investment mixes as they age to make sure they have less volatility as their retirement approaches.

GOAL BASED INVESTMENT PLANNING

In case of a traditional investment manager or a conventional Robo advisory platform, the investors are likely answer a standard risk questionnaire. It often starts with some questions to assess the risk profile and along with the amount they wish to invest and the investment horizon. But these questionnaires measure what kind of risk taker the investors think they are, not really what they should do to achieve their desired investment goals.

It's one thing to invest. It's another thing to invest with a goal in mind. For financial planning and investment advisory, Goal Planning is essential as it actually determines how much risk the investors should take. For Instance, an aggressive investor would like to save for a short term goal; we should rather recommend investments in securities bearing minimal risk. Each investor can have multiple goal, these are not just a logical classification but should be treated as a separate subaccount as it would have different portfolios and investment allocation suitable for each goal.

First point to start with; what kind of goals would the investors like to save for like building a retirement corpus that lasts for the rest of their life or a rainy-day fund for emergencies. We broadly classify these in 4 broad types;

- Retirement Fund
- Emergency Fund
- Specific Goal
- Wealth Creation

Each goal has different liquidation assumptions, so it is important for the investors to select the one that most closely matches with their real situation. Once the goal is known, the next thing to consider is how long investors will remain invested, as well as the withdrawal plan for that goal. Is it a goal that investors plan to cashing out in 10 years in lump sum to purchase a dream house, or a Retirement goal, where the investors will spend funds over a number of years rather than in one lump sum withdrawal. That’s the nature of a nest egg; where an investor withdraws a monthly income during retirement year till the expected life. The investment horizon and withdrawal pattern makes a big difference in the kind of recommendations a system should generate.

If the investors don’t have a specific investment horizon or target amount in mind, the information like age and investment horizon can be considered in a general Investing goal which can be called as ‘Wealth Creation’. Another important goal different from the rest is a ‘Rainy Day Fund’ for emergency, which in fact the first goal any investor should save for to secure a small corpus which will be needed in any contingency.

For retirement and emergency fund, one should consider monthly family expenditure to estimate what corpus would be needed by the investors. A corpus good enough to 6 to 8 months of monthly household expense should be the target investment amount for an emergency fund. Based on the current monthly expense, we can estimate the amount investors need to withdraw from the corpus on a monthly basis post retirement, till a defined life expectancy. The amount required to fulfill a goal is calculated using a time value of money considering the expected inflation rate as a discount factor, once the current value required fulfilling the goal is known.

With the information about the time horizon, target corpus and the amount the investors would like to invest which includes both a lump sum & systematic investment; we can determine an optimal portfolio the customers should invest in to reach the desired investment objective using one of the technique from a host of statistical model.

FACTOR BASED INVESTMENT ALLOCATION

Historically, financial advisors used to classify and estimate expected return and risk associated with only asset classes and it is used to propose an appropriate allocation based on return expectation and risk bearing capabilities of the investors. While there are many factors which affect the portfolio performance and risk, it could also include a wide variety of macroeconomic factors such as GDP, Interest rate, Inflation etc. but these cannot be used to allocate investments. The other factors are security specific which can help us, seek a range of goals from generating returns, reducing risk, to improving diversification. Factors classification can be based on the Asset Class, Product type or Market Exposure.

Global markets are made up of dozens of asset classes and millions of individual securities. Factors can be thought of as a group of securities with similar characteristics. It is a foundation of investing, just as nutrients are the foundations of the food we eat. We need carbohydrates and protein to power through the day, which we can find in different foods like bread, milk, and fruit. Putting together a balanced diet means understanding what nutrients are contained in our food, and choosing the mix that best supports our body's needs. Similarly, knowing the factors that drive returns in your portfolio can help us to choose the right mix of assets and strategies for specific needs of the investors.

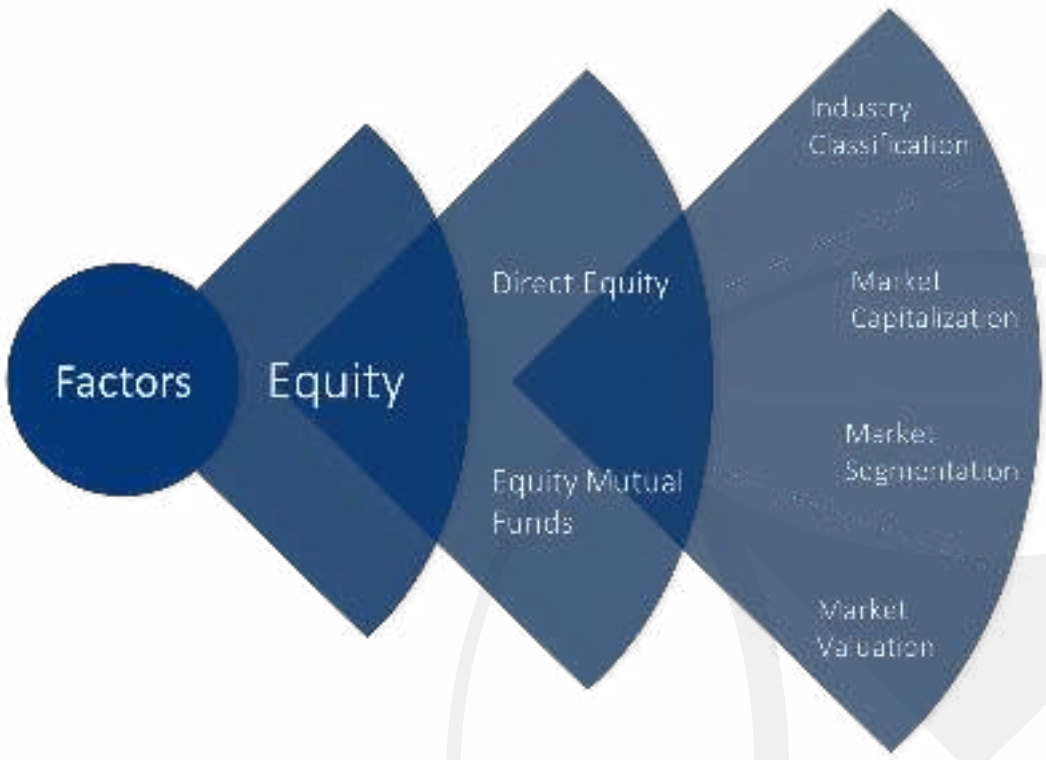


Figure1- Indicative Factor Allocation

The foundation of this approach came from a work of Eugene Fama & others, around market efficiency and multi-factor models which was recognized with a Nobel Prize in 2013.

Based on an extensive research, It was concluded that some factors earn additional returns because they involve bearing additional risk, hence your allocation to a factor is important to build portfolios that better suit individual needs; just as knowing the nutrients in your food can help your body perform. Let's understand some factors which we can use based on the profile of the investors;

Asset Classification			
Equity	**Fixed Income**	**Money Market**	**Alternate**
Moderate to high risk securities, suitable for highly & moderately aggressive & long term investors	Low to moderate risk securities, suitable for less aggressive or conservative, long term investors	Very low on risk and suits the investors averse to any risk or with short term investment horizon	Highly volatile & illiquid, suitable for aggressive investors with long term investors horizon due to higher risk

Price to Book Value		Market Classification	
High	**Low**	**Small Cap**	**Large Cap**
Known as momentum stocks, suitable for investor with short investment horizon	Known as value stocks, suitable for investors with longer investment horizon	Small cap stocks tend to outperform and bear higher risk, suitable for aggressive investors	Large cap stocks tend to underperform and less risky, suitable for less aggressive investors

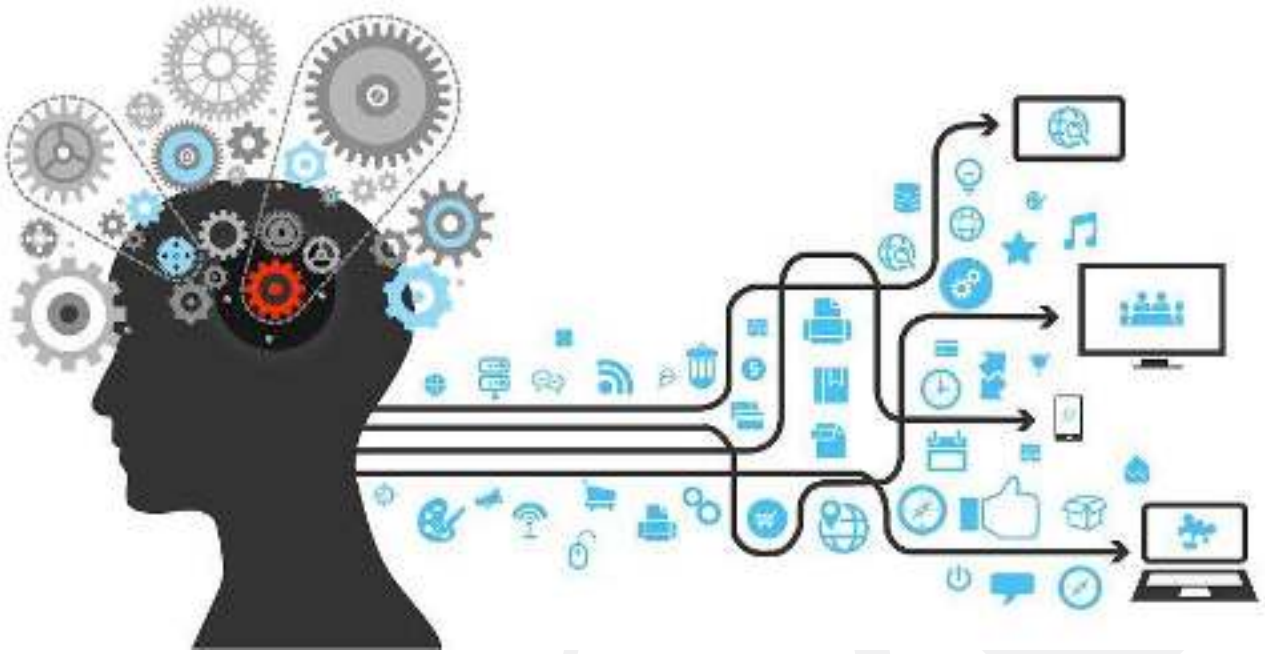
Market Exposure		Liquidity	
Emerging	**Developed**	**Less Liquid**	**Highly Liquid**
Securities exposed to emerging market are outperforms & highly volatile, suitable for aggressive investors	Securities exposed to emerging market may underperforms but are stable, suitable for aggressive investors	Less liquid securities are risky and unsuitable for conservative investor or investor with shorter investment horizon	Less liquid securities are less risky and suits for conservative investor or investor with shorter investment horizon

Figure 2- Factors & Suitable Investor Profiles

So, if there is an aggressive investor with a long-term investment horizon, we should allocate more funds to securities with Lower Price to Book Value, called as value stocks and if the investment horizon is shorter then we increase allocation to Momentum Stocks i.e. stocks with higher price to Book value since value stocks are expected to perform better in long run compared to momentum stocks which perform well in the shorter term. Similarly, we allocate more funds to small cap securities or securities exposed to emerging market for an aggressive investor as these are expected to perform better and bear higher risk than their counterparts.

Factor investing leverages advancements in today's data and technology to deliberately seek these historical return drivers in portfolios. Understanding how factors work can help you capture their potential for excess return and reduced risk, just as leading institutional investors and active fund managers have done for decades

Until now, passive investing has focused on only one factor i.e. asset allocation. The only way the investors could get access to factor based allocation was through active investment management. Factor based approach in our Robo advisory platform promises to open a new way for investing by allowing investors to access factors through passive vehicles and allocate their funds to solidly grounded & proven factors like Industry classification, Market Segmentation, Market valuation and Market Capitalization etc.



PORTFOLIO OPTIMIZATION

The allocation to selected stock is an the most important problem which needed to be addresses here, the objective is to determine the optimal mix of our chosen asset classes by using Mean-Variance Optimization, the foundation of Modern Portfolio Theory. The output of the optimization is a collection of portfolios that generate the maximum return at each level of targeted risk, or equivalently, minimize the level of risk for a specific expected return. Collectively these portfolios form the mean - variance efficient frontier.

Investment Allocation Assumptions

Mean-variance optimization (MVO) requires, as inputs, estimates of each asset class's expected return & volatility and the pair wise correlations between assets classes. The model alone can give you a portfolio highest return at minimum volatility which may not be in line with the different types of investors with varying risk appetite and return expectation. It model is typically sensitive to input parameters and tends to produce concentrated and unintuitive portfolios. To overcome the difficulty of applying MVO in practice, Fischer Black and Robert Litterman proposed the Black-Litterman model while working at Goldman Sachs (Black & Litterman, 1992). Their model applies a technique that derives expected return parameters from equilibrium allocations and specific views of manager and investors which are called as 'Constraints'. For Instance, given below are the constraints defined for different types of investors,

Highly Aggressive				
Asset Class	**Product Classes**	**Minimum (%)**	**Target (%)**	**Maximum (%)**
Equity	Direct Equity	40	45	
Equity	ETF & MF - Equity	40	50	60
Fixed Income	Government Bonds	0	0	0
Fixed Income	ETF & MF - Fixed Income	0	0	0
Money Market	Treasury Bills	0	0	0
Money Market	Money Market Funds	0	5	10
Money Market	Fixed Deposits	0	0	0

Risk Averse				
Asset Class	**Product Classes**	**Minimum (%)**	**Target (%)**	**Maximum (%)**
Equity	Direct Equity	0	0	0
Equity	ETF & MF - Equity	0	0	0
Fixed Income	Government Bonds	15	20	25
Fixed Income	ETF & MF - Fixed Income	35	40	55
Money Market	Treasury Bills	10	15	20
Money Market	Money Market Funds	10	15	20
Money Market	Fixed Deposits	5	10	15

Figure 3- Indication Allocation by Investor Profile

The Black-Litterman model provides a flexible framework to express views about asset class returns, where we can input our specific views as an indication allocation. The above section list an indicative allocation to Asset Classes and product types for 2 different types of investors, one being highly aggressive while the other is risk averse. These constraints ensure that resulting portfolio is in line with the allocation defined by the Investment Manager for a specific profile of the investor derived from their responses to the risk questionnaire, profile and goals which evaluates their ability and willingness to take risk.

The model also allows the changes in the allocation by each investor wherein they can not only change allocation to the asset & product classification but also change their allocation to any other factors defined above, helping us to capture and optimize a portfolio based on the specific views of the investors making each recommended portfolio unique.

PORTFOLIO CONSTRUCTION

The expected return of the portfolio is a weighted average of the expected returns of the individual asset classes, μ, with the weights given by the portfolio allocations, w. The variance of the portfolio depends on the variances of the individual securities, but also on how they co-vary with one another, collectively captured by the covariance matrix, ∑. To identify mean-variance efficient portfolios we solve the following quadratic programming problem,

$$\max_{w} \mu' \cdot w \text{ st. } w' \cdot \Sigma \cdot w = s^2,\ w' \cdot i = 1,\ LB \le w \le UB$$

where:

μ denotes the asset class expected returns (N x 1),

Σ denotes the asset class covariance matrix (N x N),

w denotes the asset class weights (N x 1),

i is a vector of ones (N x 1)

s is the target portoflio volatility

LB and UB are the minimum and maximum asset allocation constraints (each N x 1)

Figure 4- Formula for Black Litterman Model

To construct estimates of each securities expected return, we can use the Black-Litterman model to blend expected returns from the Capital Asset Pricing Model (CAPM). The CAPM is a simple, one-factor model which predicts that the expected return of each asset class is proportional to its beta relative to the market portfolio.

The CAPM was recognized with a Nobel Prize in 1990, and remains the cornerstone of modern finance models. Professors Eugene Fama and Kenneth French (1992, 1993) demonstrated that the CAPM provides an incomplete description of expected returns across different factors which led to the introduction of multi-factor models. Building on these insights, we add a risk premium that can vary with different factors. For Instance, Securities exposed to Emerging Markets would be with additional risk premium as they are expected to perform better and bear additional risk.

The Black-Litterman approach to constructing expected return requires three steps (Walters, 2014). First, a reverse optimization to obtain the market-implied expected returns for each asset class or factors. Second, these market-implied expected returns are blended with fund manager's or Investor's views on the various factors.

Third is covariance, a significant component of the portfolio theory, which measures how two asset moves up or down in tandem. Positive covariance means that two assets move together while the negative covariance implies that two assets move in the opposite direction. It is important to note, that when constructing a portfolio of assets, we should consider the covariance between those assets. Covariance enables us to measure the variance of the portfolio. However, when considering just one asset, then estimating the expected future return and future variance alone is sufficient. In order to have a well-diversified portfolio it is crucial to have assets with negative covariance, since when the return of one security falls, the return of the opposite security goes up and therefore it off set the potential loss.

However, covariance should not be confused with correlation coefficient, which represent the degree of how much those two assets rise or fall with respect to each other.

The volatility estimates assumes that stocks are generally riskier than bonds, even within an asset class there can be considerable variation such as risk securities exposed to emerging markets are generally riskier than the ones exposed on developed markets. Similarly, investments focused on a smaller subset of assets Small Cap Securities, Value Stocks, Industry Classification can also vary in terms of its volatility.

The correlations between stocks and bonds are negative, which means periods of decline in stock markets were associated with sharp cuts in interest rates, which caused bonds to appreciate, and bonds have been a very good diversifier for equity investments. Correlations between different types of stocks have increased recently reflecting greater global integration across economies and capital markets. Across different types of bonds, correlations with equities, range from strongly negative (US Government Bonds), to slightly positive (US corporate bonds), and very positive (Emerging Market Bonds). This trend reflects the increasing credit risk of these different types of bonds.

We use the estimates from the variance-covariance matrix of asset class returns & expected returns for each asset class as inputs to the mean-variance optimization to determine the optimal portfolio considering constraints i.e. selected allocation for different factors where we can enforce minimum and maximum allocation constraints for each factor. Unless otherwise noted, the minimum allocation constraints are set at zero in order to ensure that the optimized portfolios are long-only i.e. it should not involve any short position. Besides these allocation defined as constraints, we allow the optimizer to freely assemble portfolios from the complete set of product types.

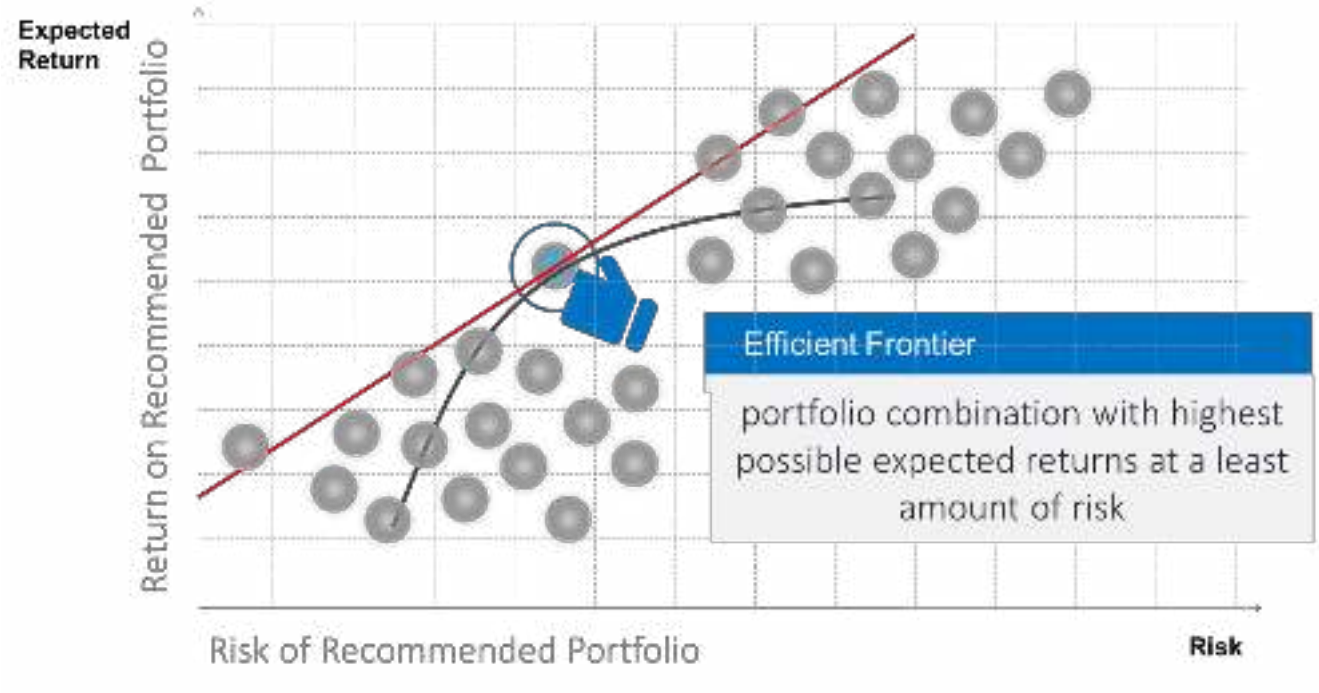


Figure 5- Efficient Frontier using Mean Variance Optimization

The efficient frontier is the curve which consists of all the portfolios that generate highest return for the given level of risk in the set of all portfolios. The efficient frontier lies between the global minimum variance portfolio or maximum Sharpe ratio portfolio. Therefore, portfolios that lie below the efficient frontier are not optimal since they generate less return for the subjected level of risk.

As the result, the portfolios that are on the efficient frontier curve tend to be more diversified. The efficient frontier can be obtained by minimizing the variance of the portfolio, subject to two primary constrains. First, all weights must add up to one, meaning we are fully invested. Second, the portfolio should provide maximum Sharpe ratio or minimum variance which is a portfolio with the least variance for any given mean return.

The investments in these portfolios can be as small as $100, which doesn't always provide sufficient cash for meaningful exposure to the entire securities being recommend or it may result in fractional quantities. As a result, for such small accounts, we use a process of holistic optimization to select the available investment only that best match the expected performance & risk parameters after rounding off or eliminating allocation to some unsuitable securities.

REBALANCING AND ONGOING MONITORING

A portfolio created using MVO based techniques will not stay optimized over time. The composition of any investment portfolio will naturally drift as capital markets move and certain holdings outperform others. This typically results in two adverse outcomes,

1. Portfolio risk increases as the equity portion of the portfolio grows beyond its original allocation, and
2. Allocations become sub-optimally mixed

To maintain the intended risk level and asset allocations, a portfolio must be periodically rebalanced back to its original targets. Sophisticated algorithms are required to optimize rebalancing subject to tax and trading expense effects. The investor's portfolios needs to be periodically rebalances each back to the target mix in an effort to optimize returns for their intended level of risk. It is important to note that the asset allocation will typically need to be adjusted over time as the investment goals and risk tolerance may change. This also requires a review their investment plans in detail to determine whether their risk tolerance and target allocation should be updated.

CONCLUSION

Robo advisory services have the potential to provide customized investment tools to individual investors at a relatively low cost at the same time, significantly mitigating behavioral finance biases. The concept of mean variance optimization may be primitive but still widely used with variations such as Black Litterman Model, Fama – French Multi Factor Model, Capital Asset Pricing Model to optimize a bespoke portfolio based on specific views and assumption of advisors and investors.

The model works on various assumptions accommodating advisor's view to select allocation based on various Factors, a custom classification by variety of characteristics of a security. While it may look simpler to end user, this approach involves creation all possible combination on portfolio allocation and select a portfolio with least risk at a given level of expected return considering allocation to all factors in mind. The expected return are estimated using CAPM where the advisor can add a risk premium based on different factors considered while the risk is denominated in the form of covariance.

It can also considers specific investment goals of the investors to derive the return expectation and the quantum of risk can be taken along with various other important inputs such as Risk Profile and Investment Goals of an investor, with a sole objective to recommend bespoke investment allocation without any human intervention.

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